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Comparative Flutter Calculations for the Viggen Aircraft

Valter J. E. Stark* Saab-Scania AB, Linköping, Sweden and Dale E. Cooley† Wright-Patterson Air Force Base, Ohio

THE canard wing configuration of the Saab-Scania Viggen aircraft is interesting for the aeroelastician since threedimensional effects and interference effects should be taken into account. Two comparative flutter calculations by different methods, but based on the same ground vibration test data, have been carried through for this configuration. One of the calculations was conducted by Saab-Scania and the other by the U.S. Air Force Flight Dynamics Laboratory (FDL).

In the two calculations, which were limited to the subsonic Mach number range and symmetric oscillations, the same idealized geometric shape was used. This consists of four pairs of trapezoidal panels S_1 , S_2 , S_3 , and S_4 , which are shown in Fig. 1. The deflection was approximated by a linear combination of five elastic modes plus the rigid translation and pitch modes. The former are characterized as follows:

1)	Wing bending	with frequency ω_i'
2)	Body bending	with frequency $1.40\omega_i$
3)	Engine mode	with frequency $1.86\omega_i'$
4)	Wing torsion	with frequency $2.48\omega_I'$
5)	Motion in wing plane	with frequency $2.59\omega_I^2$

The elastic modes contain contributions from control-surface deflections; but for the low-order modes considered, these are not significant ($\omega'_1 = 8.63 \text{ Hz}$).

Since the aerodynamic control points do not coincide with the ground vibration test points and since the streamwise slope of the deflection is also needed, interpolation was required. In the FDL calculation, this was achieved by using a

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grid of spanwise and chordwise lines connecting node points at which modal deflections were specified. The deflections were interpolated along the spanwise lines to each station at which aerodynamic control points lie, and then interpolated chordwise to the aerodynamic control point. An interpolation formula was used to determine a cubic polynomial function for the approximation. In cases for which the modal deflection is specified at more than four points spanwise or chordwise, the approximation is a piecewise-continuous cubic polynomial.

In the SAAB calculation, linear combinations of given functions of two variables were fitted by the method of least squares to the measured modal deflections.1 One combination was determined for each panel. The given functions are products of chordwise and spanwise factors. These are special polynomials with vanishing second- and third-order derivatives at free panel edges, which yields appropriate behavior of the resulting analytic modes. Control-surface deflections can be treated if needed by including discontinuous deflection functions (see Fig. 11 of Ref. 1).

The aerodynamic forces were obtained in the FDL calculation by means of the Doublet Lattice Method² and a computer program based on Ref. 3. The boxes considered in this method are formed by the common area of chordwise and spanwise strips. The panels S_1 , S_2 , S_3 , and S_4 contained 6, 6, 4, and 6 chordwise strips and 5, 10, 8, and 6 spanwise strips, respectively. The spanwise strips are formed by constant percent chord lines. The box dimensions decrease in the outboard direction to keep the box aspect ratio approximately constant and near unity. The resulting number of boxes on one half of the configuration amounted to 158.

In early flutter calculations⁴ at Saab-Scania for the Viggen aircraft, the aerodynamic forces were generated by computer programs based on the Lifting Line Element principle, which was discussed and exemplified in Ref. 5 and which also forms the basis of the Doublet Lattice Method. In this calculation, however, a new Fortran program system based on the Polar Coordinate Method⁶ has been used. The jump in the advanced velocity potential⁷ is approximated in this method by a linear combination of given potential jumps. Like the functions in the combination for the deflection, these jumps are partly products of simple chordwise and spanwise factors (integrals of Birnbaum-Glauert functions) and partly special jumps.8 The coefficients of the special jumps, which correspond to the discontinuous deflection functions, are known, whereas those of the simple jumps are to be solved from a set of linear equations. The matrix of this set is obtained by considering the velocity field that corresponds to each given potential jump and by calculating the normal component of the field at appropriate control points.

This calculation is performed by subtracting the kernel function singularity by means of a first-order polynomial, which at least in the steady case can be said to be a tangent plane9 to the potential jump, and by employing polar integration variables. The formulation implies that the normal velocity component does not ordinarily appear as a difference between large numbers, 9 and that those integrals, which must be evaluated numerically, receive well-behaved integrands.9 The velocity field that corresponds to a given potential jump is independent of the deflection mode, and the matrix of the

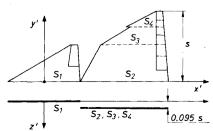


Fig. 1 Idealized Viggen configuration.

^{*}Research Scientist, Aerospace Division. Member AIAA.

[†]Technical Manager, Aeroelastic Group, U.S. Air Force Flight Dynamics Laboratory. Member AIAA.

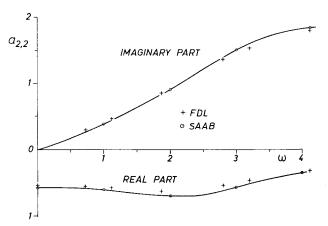


Fig. 2 Aerodynamic coefficient corresponding to the body bending mode, Mach number M = 0.7.

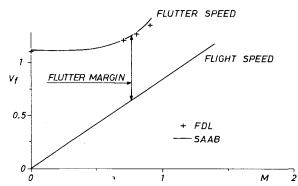


Fig. 3 Normalized flutter speed for sea-level density and flight speed for sea-level temperature.

linear equations can therefore be stored on file for later use in combination with arbitrary modes.

The aerodynamic coefficients obtained in this application are referred to the freestream dynamic pressure $\rho U^2/2$, a reference length L equal to 1.41 times the semispan s, and a reference area S=0.84 L^2 . One of the coefficients is plotted in Fig. 2 vs the reduced frequency $\omega = \omega' L/U$; ω' is the circular frequency. The figure shows that the results of the FDL calculation and the SAAB calculation are in good agreement.

The flutter speed was determined in the FDL calculation by the p-k method and in the SAAB calculation by the p method. Decay rates calculated by these methods may be more reliable 10 than that obtained by the V-g method, but all three methods yield the same result for the flutter speed. The SAAB calculation was carried through by using a new Fortran program system called AEREL which includes the subprograms HCOEFF, PCP, and STAB. HCOEFF determines analytic deflection modes and mass matrices, PCP generates aerodynamic matrices by the Polar Coordinate Method, and STAB solves the eigenvalue problem by various methods. In case of the p method or the p-k method, the eigenvalue problem is solved by iteration. In each iteration step, a new eigenvalue is calculated by employing an aerodynamic matrix corresponding to the eigenvalue obtained in the preceding step and a program for solution of eigenvalues of a general complex matrix. 11 Approximate polynomials in $i\omega$ with real coefficients are used in STAB in each step for generation of the aerodynamic matrix. Each subprogram can read input data from a file generated by another subprogram.

Results from the two calculations for the flutter speed are plotted vs Mach number M for standard-day sea-level density in Fig. 3. The + signs represent the FDL results and the upper curve the SAAB results for the normalized flutter speed $v_f = U_f/\omega_I'L$. The agreement is seen to be very satisfactory. The results for the flutter frequency also agree very well. They

decrease monotonically from 1.36 ω'_1 at M=0 to 1.30 ω'_1 at M=0.9. It is essentially the first two elastic modes that form the flutter mode.

The straight line in Fig. 3 represents the normalized flight speed $v = U/\omega_i'L$ for standard-day sea-level temperature. Comparing the upper curve to this, we see that the flutter speed is much greater than the flight speed or that a large flutter margin exists. The minimum value of this is about 77% of the flight speed in the Mach number range considered.

In conclusion it may be said that it is satisfying that the two calculations have produced agreeing results. The methods and programs employed in both of them therefore seem reliable. It is also satisfying that the calculations predict a large flutter margin. This corroborates previous results. It must be added, however, that the calculations apply only to one flight condition, namely zero angle of incidence. For an increasing angle of incidence, the aerodynamic coefficients probably increase due to the developing leading edge vortices. This may reduce the flutter speed, but the remaining margin probably is large and sufficient.

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Leading-Edge Vortex Effect on the Flutter Speed

Valter J. E. Stark*
Saab-Scania AB, Linköping, Sweden

BECAUSE of the vortices that develop at a swept leading edge for increasing angle of incidence α , the slope of the local lift coefficient curve increases at outboard wing stations. The flutter speed, therefore, decreases. This has been shown

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^{*}Research Scientist, Aerospace Division. Member AIAA.